On gauge invariance and spontaneous symmetry breaking

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Abstract. We show how the widely used concept of spontaneous symmetry breaking can be explained in causal perturbation theory by introducing a perturbative version of quantum gauge invariance. Perturbative gauge invariance, formulated exclusively by means of asymptotic fields, is discussed for the simple example of Abelian $U(1)$ gauge theory (Abelian Higgs model). Our findings are relevant for the electroweak theory, as pointed out elsewhere.

1. Introduction

It is quite a common assumption that scalar QED with massive photons is not a gauge theory in the usual sense, because the introduction of a mass term in the Lagrangian for the gauge field violates the classical gauge invariance of the theory. Therefore, a ‘Higgs’ field with a nonvanishing vacuum expectation value is usually coupled to the photon which then acquires a mass [1]. Proceeding in this way, the local $U(1)$ invariance is not absent, but ‘hidden’.

It is the aim of this paper to demonstrate how massive gauge theories can be described in the framework of causal perturbation theory [2] by means of a perturbative version of quantum gauge invariance (25). Perturbative gauge invariance has the advantage that it provides a powerful tool for the actual construction of the theory. We will demonstrate this for the Abelian Higgs model in section 4§. Starting from a cubic coupling $\sim A_\mu A^\mu \phi$, gauge invariance of first order demands the introduction of scalar ghost fields $u, \tilde{u}$ and of an additional unphysical scalar field $\Phi$ and fixes most of the cubic couplings. Then, gauge invariance to second order determines the remaining cubic couplings and requires additional quartic ones. One has to go to third order to fix the quartic couplings completely. The resulting couplings contain the Higgs potential which, however, comes out as a quartic polynomial in the original asymptotic scalar field $\phi$ with vanishing vacuum expectation value $\langle \phi \rangle = 0$. This means that gauge invariance leads us directly to the final theory ‘after spontaneous symmetry breaking’. Although we can see the symmetry breaking in the double-well potential at the end, it plays no direct role in the construction: perturbative gauge invariance alone does the job.

The method works beautifully in the more complicated situations of the electroweak theory, as pointed out in detail elsewhere [9, 10].

§ We would like to thank Bert Schroer for posing this problem.
2. Gauge invariance for massive gauge fields

2.1. Causal perturbation theory

Our work is best done in the framework of causal perturbation theory, which has its roots in a classical paper by Epstein and Glaser [2]. In this approach the $S$-matrix is constructed inductively order by order in the form

$$S(g) = 1 + \sum_{n=1}^{\infty} \frac{1}{n!} \int d^4x_1 \ldots d^4x_n T_n(x_1, \ldots, x_n) g(x_1) \ldots g(x_n)$$

(1)

where $g(x)$ is a tempered test function that switches the interaction. The first order (e.g. for QED)

$$T_1(x) = i e : \bar{\Psi}(x)\gamma^\mu \Psi(x) : A_\mu(x)$$

(2)

must be given in terms of the asymptotic free fields. It is a striking property of the causal approach that no ultraviolet divergences appear, i.e. the $T_n$’s are finite and well defined up to finite normalization terms. The adiabatic limit $g(x) \to 1$ has been shown to exist in purely massive theories at each order [2].

The crucial point in the causal formulation of perturbation theory is that the usual formal definition of the $T_n$ via simple time ordering

$$T_n(x_1, \ldots, x_n) = T\{T_1(x_1), \ldots, T_n(x_n)\}$$

$$\equiv \sum_P \Theta(x_{\pi_1}^0 - x_{\pi_2}^0), \ldots, \Theta(x_{\pi_{n-1}}^0 - x_{\pi_n}^0) T_1(x_{\pi_1}), \ldots, T_1(x_{\pi_n})$$

(3)

(4)

where the sum runs over all $n!$ permutations, contains ultraviolet divergences, therefore there must be an error in the derivation. Epstein and Glaser [2] proceeded more carefully and introduced the following $n$-point distributions:

$$A'_n(x_1, \ldots, x_n) = \sum_{P_2} \tilde{T}_n(X) T_{n-n_1}(Y,x_n)$$

(5)

$$R'_n(x_1, \ldots, x_n) = \sum_{P_2} T_{n-n_1}(Y,x_n) \tilde{T}_{n_1}(X)$$

(6)

where the sums run over all partitions

$$P_2 : [x_1, \ldots, x_{n-1}] = X \cup Y \quad X \neq \emptyset$$

(7)

into disjoint subsets with $|X| = n_1$, $|Y| \leq n - 2$. Assuming by induction that $T_1, \ldots, T_{n-1}$ are known, then $A'_n$ and $R'_n$ can be calculated. One also introduces

$$D_n(x_1, \ldots, x_n) = R'_n - A'_n$$

(8)

If the sums are extended over all partitions $P_2^0$, including the empty set $X = \emptyset$, we obtain the distributions

$$A_n(x_1, \ldots, x_n) = \sum_{P_2^0} \tilde{T}_n(X) T_{n-n_1}(Y,x_n)$$

(9)

$$= A'_n + T_n(x_1, \ldots, x_n)$$

(10)

$$R_n(x_1, \ldots, x_n) = \sum_{P_2^0} T_{n-n_1}(Y,x_n) \tilde{T}_{n_1}(X)$$

(11)

$$= R'_n + T_n(x_1, \ldots, x_n).$$

(12)
These two distributions are not known by the induction assumption because they contain the unknown $T_n$. Only the difference

$$D_n = R'_n - A'_n = R_n - A_n$$

is known. We stress the fact that all products of distributions here are well defined because the arguments are disjoint sets of points so that the products are tensor products of distributions.

One can determine $R_n$ or $A_n$ separately by investigating the support properties of the various distributions. Causality of the $S$-matrix requires that $R_n$ is a retarded and $A_n$ is an advanced distribution [2, 3]

$$\text{supp } R_n \subseteq \bar{V}^{+}(x) \quad \text{supp } A_n \subseteq \bar{V}^{-}(x)$$

Hence, by splitting the causal distribution (13) one obtains $R_n$ (and $A_n$), and $T_n$ then follows from (10) (or (12)). The $T_n$’s so obtained are well defined time-ordered products. Local terms with support $(x_1 = \cdots = x_n)$, originating from a certain ambiguity in the splitting procedure, might spoil the symmetry of the $T_n$’s in $x_1, \ldots, x_n$, but this minor problem can be removed by subsequent symmetrization.

To carry out the splitting process, we write (13) in normally ordered form and split the numerical distributions $d_n^O(x)$, where $x = (x_1 - x_n, \ldots, x_{n-1} - x_n)$

$$D_n(x_1, \ldots, x_n) = \sum_O d_n^O (x_1 - x_n, \ldots, x_{n-1} - x_n) : O(x_1, \ldots, x_n) :$$

where $: O :$ is a normally ordered product of external field operators (Wick monomial). It is a consequence of translation invariance that $d_n^O(x)$ only depends on relative coordinates.

The only nontrivial step in the construction of well defined time-ordered products is the splitting of a numerical distribution $d$ with support in $\bar{V}^{+} \cup \bar{V}^{-}$ into a distribution $r$ with support in $\bar{V}^{+}$ and a distribution $a$ with support in $\bar{V}^{-}$. In causal perturbation theory the usual formal time-ordered products with subsequent renormalization are replaced by this conceptually simple and mathematically well defined procedure. In fact the problem of distribution splitting was already solved in a general framework by the mathematician Malgrange in 1960 [4]. Epstein and Glaser [2] used his general result for the special case of relativistic quantum field theory. A simple solution for the splitting problem can be found in [3].

2.2. Gauge invariance for massive QED

Since the above construction of the perturbative $S$-matrix only uses the asymptotic free fields, we are looking for a formulation of quantum gauge invariance in terms of these fields.

We first discuss the simple case of quantum electrodynamics with massive photons. Let

$$Q \overset{\text{def}}{=} \int d^3 x \left( \partial_{\mu} A^\mu(x) + m \Phi(x) \right) \bar{\psi}_{\beta}(x) \gamma_{\beta} \psi(x)$$

be the generator of (free) gauge transformations, called gauge charge for brevity. $A_\mu$ is the gauge potential in the Feynman gauge, $\psi, \bar{\psi}$ are fermionic ghost fields and $\Phi$ is a neutral
scalar, satisfying the well known commutation relations
\[
\begin{align*}
[A^{(\pm)}_\mu(x), A^{(\mp)}_\nu(y)] &= ig^{\mu\nu}D_m^{(\mp)}(x-y) \\
[u^{(\pm)}(x), u^{(\mp)}(y)] &= -iD_m^{(\mp)}(x-y) \\
[\Phi^{(\pm)}(x), \Phi^{(\mp)}(y)] &= -iD_m^{(\mp)}(x-y)
\end{align*}
\]
and all other (anti)commutators vanish. All these fields fulfill the Klein–Gordon equation with the same mass \(m\). In order to see how the infinitesimal gauge transformation acts on the free fields, we calculate the (anti)commutators

\[
\begin{align*}
[Q, A_\mu] &= i\partial_\mu u \\
\{Q, \Phi\} &= imu \\
\{Q, u\} &= 0 \\
\{Q, \bar{u}\} &= -i\partial_\mu A^\mu - im\Phi \\
[Q, \Psi] &= 0.
\end{align*}
\]

Then we have

\[
\begin{align*}
[Q, T_1(x)] &= -e :\bar{\Phi}\gamma^\mu\Phi : \partial_\mu u \\
&= i\partial_\mu(i\epsilon :\bar{\Phi}\gamma^\mu\Phi : u) = i\partial_\mu T_1^\mu(x).
\end{align*}
\]

Assuming that the operation of commuting with \(Q\) commutes with time ordering, we obtain

\[
[Q, T_n(x_1, \ldots, x_n)] = i\sum_{l=1}^n \partial^\mu_l T^\mu_{n/l}(x_1, \ldots, x_n) = \text{(sum of divergences)}
\]

where \(T^\mu_{n/l}\) is a mathematically rigorous version of the time-ordered product

\[
T^\mu_{n/l}(x_1, \ldots, x_n) = T(T_1(x_1) \ldots T^\mu_1(x_l) \ldots T_1(x_n))
\]

constructed by means of the method of Epstein and Glaser [2] described above. We define (25) to be the condition of gauge invariance [3]. For a fixed \(x_l\) we consider from \(T_n\) all terms with the external field operator \(A_\mu(x_l)\)

\[
T_l(x_1, \ldots, x_n) = :t^\mu_l(x_1, \ldots, x_n)A_\mu(x_l) : + \cdots
\]

(the dots represent terms without \(A_\mu(x_l)\)). Then gauge invariance requires

\[
\partial^\mu_l [t^\mu_l(x_1, \ldots, x_n)u(x_l)] = t^\mu_l(x_1, \ldots, x_n)\partial^\mu_l u(x_l)
\]

or

\[
\partial^\mu_l t^\mu_l(x_1, \ldots, x_n) = 0.
\]

It is an interesting observation that although the photon is massive, it is not necessary to introduce a ‘Higgs’ field to give an explanation for this fact.

3. Unitarity

Equation (29) is the usual gauge invariance condition as in the massless case [3], where no scalar \(\Phi\) is needed. Moreover, \(\Phi\) and the ghost fields do not couple at all. Therefore, we have to explain why the unphysical fields have been introduced. The reason is that it allows us to prove the unitarity of the \(S\)-matrix on the physical Hilbert space \(H_{\text{phys}}\), which is a subspace of the Fock–Hilbert space \(F\) also containing the unphysical ghosts and scalars.

The basic property for unitarity is the nilpotency of the gauge charge \(Q\)

\[
Q^2 = \frac{1}{2} [Q, Q] = 0
\]

and the Krein structure on the Fock–Hilbert space [5–8]. Then the physical Fock space can be expressed by the kernel and the range of \(Q\), namely

\[
H_{\text{phys}} = \ker Q \ominus \text{ran } Q = \ker [Q, Q^+].
\]
This can be seen most easily by realizing the various field operators on a positive definite Fock–Hilbert space $F$:

$$\begin{align*}
A^\mu(x) &= (2\pi)^{-3/2} \int \frac{d^3k}{(2\omega)} \left( \epsilon^\mu_\lambda(k)a_\lambda(k)e^{-ikx} \pm (\epsilon^\mu_\lambda(k)a^\dagger_\lambda(k)e^{+ikx} \right) \\
\omega &= \sqrt{k^2 + m^2}
\end{align*}$$

(32)

where $\epsilon^\mu_\lambda$ are four polarization vectors satisfying

$$\begin{align*}
\epsilon^\mu_0 &\equiv \frac{k^\mu}{m} \\
3 \sum_{\lambda=0}^3 g_{\lambda\lambda}\epsilon^\mu_\lambda &\equiv g^{\mu\nu} \epsilon^\nu_\lambda = \epsilon^\mu_\lambda
\end{align*}$$

(33)

(34)

and we have a minus sign for $\lambda = 0$ in (32) to be consistent with Lorentz invariance. A similar asymmetry occurs in the ghost sector

$$\begin{align*}
u(x) &= (2\pi)^{-3/2} \int \frac{d^3k}{\sqrt{2\omega}} \left( c_2(k)e^{-ikx} + c_1(k)^+e^{ikx} \right) \\
tilde{\nu}(x) &= (2\pi)^{-3/2} \int \frac{d^3k}{\sqrt{2\omega}} \left( -c_1(k)e^{-ikx} + c_2(k)^+e^{ikx} \right).
\end{align*}$$

(35)

(36)

All creation and annihilation operators satisfy the usual commutation relations. Then the proof of unitarity is exactly the same as in [5, 6].

We wish to emphasize that we are not forced to represent the gauge potential in the Feynman gauge as in (32). If we did not do so, the unphysical particles would acquire a mass depending on the gauge fixing parameter. Furthermore, in the case of a massless photon, the above considerations remain valid with a little exception: The unphysical scalar field $\Phi$ would no longer appear in the gauge charge $Q$, therefore it would become physical and its mass could be chosen arbitrarily, or the field could be removed from the theory.

The full power of the above concept shows up if non-Abelian gauge fields are introduced (e.g. in electroweak theory [9, 10]). The example which follows shows some essential features of the more complicated discussion in case of the electroweak theory. For simplicity, in section 4 we will demonstrate how perturbative gauge invariance fixes all couplings in the case of an Abelian theory. In a sense, we will derive the ‘Higgs’ potential.

4. The Abelian Higgs model

4.1. Gauge invariance at first order

Consider the simple case of classical Abelian $U(1)$ gauge theory [11], given by the Lagrangian

$$\begin{align*}
\mathcal{L} &= (\partial_\mu + igB_\mu)\varphi^+ (\partial^\mu - igB^\mu)\varphi + \mu^2\varphi^+\varphi - \lambda(\varphi^+\varphi)^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\
F^{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu.
\end{align*}$$

(37)

(38)

If we assume that the scalar field $\varphi$ develops a vacuum expectation value $|\langle 0 |\varphi |0 \rangle| = v/\sqrt{2} = (\mu^2/2\lambda)^{1/2}$, then in the unitary gauge we obtain the Lagrangian

$$\begin{align*}
\mathcal{L} &= \frac{1}{2} (\partial_\mu \phi)^2 - \frac{1}{2} m_H^2 \phi^2 - \frac{1}{2} (\partial_\mu A_\nu - \partial_\nu A_\mu)^2 + \frac{1}{2} m^2 A_\mu A^\mu + g^2 vA_\mu A^\mu \\
&+ \frac{1}{2} g^2 A_\mu A^\mu \phi^2 - \lambda \phi^3 - \frac{1}{2} \lambda \phi^4
\end{align*}$$

(39)

$$m = gv \quad m_H = \sqrt{2}\mu$$

(40)
where $\phi$ is Hermitian and $(A_\mu, (v + \phi(x))/\sqrt{2})$ are obtained from $(B_\mu, \varphi)$ by a local $U(1)$
transformation

$$\varphi(x) = \frac{1}{\sqrt{2}}(v + \phi(x))e^{i\xi(x)/v}, \quad B_\mu(x) = A_\mu(x) + \frac{1}{gv}\partial_\mu\xi(x). \quad (41)$$

Now we derive the whole quantum theory in a totally different way. Our starting point is
the first-order coupling $\sim A_\mu A^\mu \phi$ of the physical fields $A_\mu$ and $\phi$ with masses $m$ and $m_\mu$, respectively. Furthermore, we introduce the unphysical scalar field $\Phi$ which appears in the
gauge charge $Q$. The latter is still given by (17) and the guiding principle is the operator
gauge invariance (25). Then a general ansatz for the first-order coupling, containing only
trilinear terms in the free fields and leading to a renormalizable theory, is

$$T_1(x) = igm : [A_\mu A^\mu \phi + \alpha A_\mu A^\mu \Phi + \beta_1 u u \Phi + \beta_2 u \Phi]* = 1$$

We calculate $d_Q T_1 \equiv [Q, T_1]$ and obtain

$$d_Q T_1 = - g m : [2 \partial_\mu(u(A^\mu \phi + \alpha A^\mu \Phi)) + \gamma \partial_\mu(u(\phi \partial^\mu \Phi - \Phi \partial^\mu \phi)) + \gamma \mu \mu A_\mu A^\mu \Phi - 2 \alpha u \mu \partial_\mu A^\mu \Phi]$$

where we have taken out the derivatives of the ghost fields. Since $d_Q T_1$ has to be a pure
divergence, the terms which are not of this form must cancel. This fixes most of the free
parameters. We immediately obtain

$$T_1 = igm : $$

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where we have taken out the derivatives of the ghost fields. Since $d_Q T_1$ has to be a pure
divergence, the terms which are not of this form must cancel. This fixes most of the free
parameters. We immediately obtain

$$T_1 = igm : $$

and

$$d_Q T_1 = - g m : \partial_\mu \left( \frac{u(A^\mu \phi)}{m} - \frac{1}{m}(\phi \partial^\mu \Phi - \Phi \partial^\mu \phi) \right) \equiv \delta_\mu T_1^{\mu \nu}. \quad (45)$$

Obviously, the quadrilinear couplings in (39) are still missing, and $\delta_4$ is not yet fixed.
Therefore, we have to discuss gauge invariance at second and third order.

4.2. Gauge invariance at second and third order

Following the inductive construction of Epstein and Glaser, we have to first calculate the
causal distribution

$$D_2(x, y) = T_1(x)T_1(y) - T_1(y)T_1(x) = - \tilde{A}_2(x, y) + R_2^0(x, y). \quad (46)$$

The main problem is whether gauge invariance can be preserved in the distribution splitting.
Obviously, $D_2$ is gauge invariant:

$$d_Q D_2 \equiv [d_Q T_1(x), T_1(y)] + [T_1(x), d_Q T_1(y)]$$

$$= i \delta_\mu \left( T_1^{\mu \nu}(x), T_1(y) \right) + i \delta_\mu \left( T_1(x), T_1^{\mu \nu}(y) \right) \equiv i \delta_\mu T_1^{\mu \nu}(x, y) + i \delta_\mu D_2^\nu(x, y). \quad (47)$$

Since the retarded part $R_2$ agrees with $D_2$ on the forward light cone $V^+ \setminus \{ x = y \}$ and
similarly for $R_2^2$, $D_2$ gauge invariance of $R_2$ can only be violated by local terms
$\sim D^\mu \delta(x - y)$. But such local terms are precisely the freedom of normalization in the
distribution splitting. If the normalization terms \( N_2, N_{2/1}^\mu, N_{2/2}^\mu \) can be chosen in such a way that
\[
d_Q(R_2 + N_2) = i\partial_\mu(R_{2/1}^\mu + N_{2/1}^\mu) + i\partial_\mu(R_{2/2}^\mu + N_{2/2}^\mu)
\]
holds, then the theory is gauge invariant in second order. Note that the distribution \( T_2 = R_2 + N_2 - R_2' \) also fulfils (48). The local terms on the right-hand side of (48), which come from the causal splitting, are called anomalies. The ordinary axial anomalies in the standard model are of the same kind, they appear in the third-order triangle diagrams with axial vector couplings to fermions [10].

We consider the following example: in the commutator \([T_{1/1}^\mu(x), T_1(y)]\) appears the term
\[
-g^2 m : u(x)\Phi(x)[\partial_\mu\phi(x), \phi(y)]A_\nu(y)A^\nu(y) :
\]
\[
= ig^2 m : u(x)\Phi(x)A_\nu(y)A^\nu(y) : \partial_\mu D_{m\mu}(x - y).
\]

After splitting this causal distribution the Pauli–Jordan distribution \( D_m \) is replaced by the retarded distribution \( D_m^{\text{ret}} \). If we now calculate the divergence of (49), we obtain an anomaly
\[
\frac{A_1}{2} = ig^2 m : u\Phi A_\nu A^\nu : \delta(x - y)
\]
but because
\[
\partial_\mu A_\nu D_{m\mu}(x - y) = -m^2 D_{m\mu}^{\text{ret}}(x - y) + \delta(x - y).
\]
The terms with \( x \) and \( y \) interchanged lead to the same contribution. But in the causal distribution \( D_2 = [T_1(x), T_1(y)] \) the term
\[
-g^2 : A_\mu(x)\Phi(x)[\partial_\mu\phi(x), \partial_\nu\phi(y)]A_\nu(y)\Phi(y) :
\]
\[
= -ig^2 : A_\mu(x)A_\nu(y)\Phi(x)\Phi(y) : \partial_\mu \partial_\nu D_{m\mu}(x - y)
\]
appears, which has singular degree \( \omega = 0 \) [2, 3] and therefore allows a normalization term in the split distribution
\[
\partial_\mu \partial_\nu D_{m\mu}^{\text{ret}}(x - y) \to \partial_\mu \partial_\nu D_{m\mu}^{\text{ret}}(x - y) + C g^{\mu\nu} \delta(x - y).
\]

Since
\[
d_Q(: \Phi^2 A_\mu A^\mu : C\delta(x - y)) = 2iCm : u\Phi A_\mu A^\mu : \delta(x - y) + \cdots
\]
we can compensate the anomaly (50) by choosing \( C = ig^2 \). In this way we obtain the quadrilinear couplings of the theory as normalization terms in higher orders. We give here the complete list of all normalization terms for tree diagrams in second order:
\[
N_1 = ig^2 : A_\mu A^\mu \Phi^2 : \delta(x - y)
\]
\[
N_2 = ig^2 : A_\mu A_\nu \phi^2 : \delta(x - y)
\]
\[
N_3 = -ig^2 \frac{m_H^2}{4m^2} : \Phi^4 : \delta(x - y)
\]
\[
N_4 = ig^2 \left( \frac{m_H^2}{m^2} + 3\delta_4 \right) : \phi^2 \Phi^2 : \delta(x - y)
\]
\[
N_5 = ig^2 \lambda' : \phi^4 : \delta(x - y) \quad \lambda' \text{ still free.}
\]

The remaining free parameters \( \delta_4 \) and \( \lambda' \) can be determined by considering the anomalies \( \sim \delta(x - z)\delta(y - z) \) of tree diagrams in third order. They arise in the splitting of terms in
\[
D_{1/1}^\mu(x, y, z) = [T_{1/1}^\mu(x), T_2(y, z)] + \cdots
\]
where $T_{1/1}^{\mu}$ (45) gets contracted with a normalization term $N_{1-5}$ in $T_2$. Considering all anomalies $\sim: u\Phi^3:,$ gauge invariance requires

$$2\lambda' = \frac{m_H^2}{m^2} + 3\delta_4 $$

(62)

and from the anomalies $\sim: u\Phi^3:,$ we obtain

$$\delta_4 = -\frac{m_H^2}{2m^2} $$

(63)

in agreement with (39).

Besides some basic assumptions concerning simplicity (42), we have constructed the theory with the help of a guiding principle, namely perturbative quantum gauge invariance, which, after its construction, is manifest in our approach.

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References